Problem 1

Solve the equation

$$x + \sin x - 1 = 0$$

using bisection method with accuracy $\varepsilon = 10^{-4}$.

Solution

Let's find the interval where the equation has the unique root. Consider the function $f(x) = x + \sin x - 1$. It's derivative:

$$f'(x) = 1 + \cos x \geq 0$$

doesn't change the sign, so equation has at most one root.

$f(0) = -1 < 0, f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} > 0$. So the root is inside the interval $\left[0, \frac{\pi}{2}\right]$.

Let's choose $a_0 = 0, b_0 = \frac{\pi}{2}$. Number of iterations required to reach needed accuracy is 13. Values of x_n are given in the table:

n	x_n	$f(x_n)$
0	0.785398	0.492505
1	0.392699	-0.224618
2	0.589049	0.144620
3	0.490874	-0.037729
4	0.539961	0.054064
5	0.515418	0.008317
6	0.503146	-0.014670
7	0.509282	-0.003168
8	0.512350	0.002577
9	0.510816	-0.000295
10	0.511583	0.001141
11	0.511199	0.000422
12	0.511007	0.000063
13	0.510911	-0.000117

Problem 2

Find the least root of the equation

$$x^3 + 3x^2 - 1 = 0$$

with accuracy $\varepsilon = 10^{-4}$ using Newton's method.

Solution

This roots has unique root on interval $[0; 1]$. $f(0.5) = -0.125$. Now let's find the root on $[0.5; 1]$. Let $f(x) = x^3 + 3x^2 - 1$. Then $f'(x) = 3x^2 + 6x > 0$, $f''(x) = 6x + 6 > 0 \; if \; [0.5; 1]$.

$$m_1 = \min_{x\in[0.5;1]} |f'(x)| = |f'(0.5)| = 3.75$$

$$M_2 = \max_{x\in[0.5;1]} |f''(x)| = |f''(1)| = 12$$

Let's choose $x_0 = 1, |x - x^*| \leq 0.5$.

$$q = \frac{12 \cdot 0.5}{2 \cdot 2.75} = 0.8 < 1$$

So, all conditions of Newton's method holds. It is enough to do 7 iterations. Calculations are in the table:

n	x_n	$f(x_n)$
0	1	3
1	0.666666667	0.62962963
2	0.548611111	0.068040217
3	0.532390162	0.00121814
4	0.532088989	4.16958E-07
5	0.532088886	4.86278E-14
6	0.532088886	0
7	0.532088886	0