Assignment4Student
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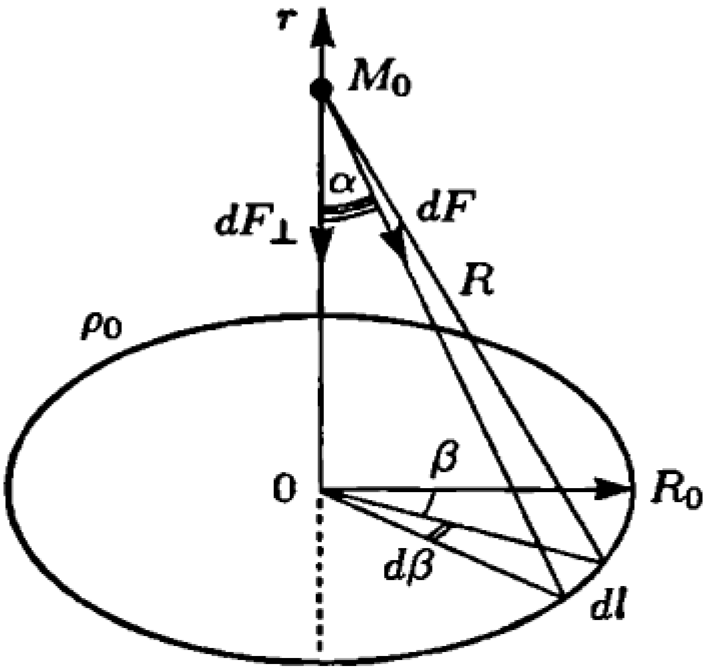
**Problem**

Construct an model of movement of point mass $M_0$ in field of gravitational force created by ring with radius $R_0$ and linear density $\rho_0$. The point moves along ring axis.

**Solution**

The forces are drawn on picture:




Let's compute gravity force between point $M_0$ and mass $dm$ contained in element of ring $dl$:

$$dF = \gamma \frac{M_0 dm}{R^2}$$

Using trigonometric functions:

$$\frac{R_0}{R} = \sin\alpha = \frac{R_0}{\sqrt{r^2 + R_0^2}}$$

$$\frac{r}{R} = -\cos\alpha = \frac{r}{\sqrt{r^2 + R_0^2}}$$

$$dm = \rho_0 dl = \rho_0 R_0 d\beta = -\rho_0 r \tan\alpha \, d\beta$$

$$dF = -\gamma \frac{M_0 \rho_0}{R^2} r \tan\alpha \, d\beta = -\gamma \frac{M_0 \rho_0}{r} \sin\alpha \cos\alpha \, d\beta$$

Let's find projection of F on axis r.

$$dF_1 = dF\cos\alpha = -\gamma\frac{M_0\rho_0}{r}\sin\alpha\cos^2\alpha\, d\beta$$

Now we must add all forces acting on point. We will do it by integration on angle:

$$F = \int_0^{2\pi} F_1 d\beta = \int_0^{2\pi} -\gamma\frac{M_0\rho_0}{r}\sin\alpha\cos^2\alpha\, d\beta = -2\pi\gamma\frac{M_0\rho_0}{r}\sin\alpha\cos^2\alpha$$
$$= -\gamma M_0 M_1 \frac{r}{(r^2 + R_0^2)^{\frac{3}{2}}}$$

where

$$M_1 = 2\pi R_0\rho_0$$

is the full mass of the ring.