Problem

The tallow in amount of 3660 kg/hour in mix with 1030 kg/hour of hot water under high pressure is moved from below to a spray column working with temperature 232^o and pressure 42.18 kg/cm^2. Under the same conditions of temperature and pressure 1870 kg/hour of water is dispersed from above and moves down as drops. Glycerin is generated due to hydrolysis, is extracted by water and so 2530 kg/hour of final extract with 12.16% of glycerin is continuously going below of the column. In the same time 4050 kg/hour of raffinate (it consists of fatty acid and 0.24% of glycerin) is going out in the top of the column. Using effective height of the column is 22m and diameter is 0.72m, equivalent content of glycerin in tallow is 8.53%, relative distribution of glycerin between water and fat is 10.32, determine changing of glycerin concentration on column height. Determine part of column needed for chemical reaction.

Solution

Let's use next conventional signs:

x – mass fraction of glycerin in raffinate

y- mass fraction of glycerin in extract

y^* - mass fraction of glycerin in extract in balance with raffinate for value x

z – mass fraction of tallow in raffinate

S – area of cross-section of column

a – interphase area per unit of colume

K – total coefficient of mass exchange

m – coefficient of distribution

k – constant of reaction speed

ρ – mass of tallow per unit of volume

h – length of column

w- mass of tallow per 1 kg of glycerin

H – effective height of column

Consider an element of column with height δh. Mass of glycerin transforming from tallow to aqueous phase is $KaS(y^* - y)\delta h$. The speed of generating glycerin is $\frac{k\rho Sz\delta h}{w}$. For balance of glycerin due to element of column we have:

$$Lx + \frac{k\rho Sz\delta h}{w} - L\left(x + \frac{dx}{dh}\delta h\right) = Gy - G\left(y + \frac{dy}{dh}\delta h\right) = KaS(y^* - y)\delta h$$

Between elementary volume and bottom:

$$\frac{Lx_0}{w} + Gy = Lx + \frac{Lz}{w} + Gy_0$$

Between phases:

$$y^* = mx$$

It follows that

$$KaSmx = KaSy - G\frac{dy}{dh}$$

Substituting this we get:

$$k\rho S\left(\frac{z_0}{w} + \frac{mG}{L}(y - y_0) - x\right)\delta h - L\frac{dx}{dh}\delta h = -G\frac{dy}{dh}\delta h$$

Multiplying this by $\left(\frac{KaSm}{LC}\right)$ and substituting x we get:

$$\frac{k\rho S^2 Ka}{LG}\left(\frac{mz_0}{w} + \frac{mG}{L}(y - y_0)\right) - \frac{k\rho S}{L}\left(\frac{KaS}{G}y - \frac{dy}{y}\right) - \left(\frac{KaS}{G}\cdot\frac{dv}{dh} - \frac{d^2y}{dh^2}\right) + \frac{KaSm}{L}\cdot\frac{dv}{dh} = 0$$

Let’s denote:

$$r = \frac{mG}{L};\quad p = k\backslash pho\frac{S}{L};\quad q = \frac{KaS}{G}(r - 1)$$

After substituting:

$$\frac{d^2y}{dh^2} + \frac{(p+q)dy}{dh} + pqy = \frac{pq}{r-1}\left(ry_0 - \frac{mz_0}{w}\right)$$

This is linear differential equation with solution

$$y_c = Ae^{-ph} + Be^{-qh} + \frac{ry_0 - mz_0/w}{r-1}$$

We should find A and B from boundary conditions:

$$h = 0 \quad x = 0$$

$$h = H \quad y = 0$$

After substitutions we have 2 equations:

$$A + B + \frac{r-1}{q}(pA - qB) + C = 0$$

$$Ae^{-pH} + Be^{-qH} + C = 0$$

Let

$$v = \frac{q + rp - p}{q} = 1 + \frac{kpG}{KaL}$$

After all calculations we get:

$$y = \frac{mz_0}{w(r-v)}\left(e^{-ph} + \frac{e^{-pH} - v}{r - e^{-qH}}e^{-qh} + \frac{ve^{-qH} - re^{-pH}}{r - e^{-qH}}\right)$$

This equation show mass fraction of glycerin depending of h.

$$L = 3880, G = 1700, y_0 = 0.188$$

$$r = 4.55; \rho = 0.0605, q = 0.202$$

$$v = 1 + \frac{250}{Ka}$$

Solving this at $H = 22$ we have:

$$Ka = 238$$

After calculations we see that chemical reaction ends at height 9m.