Problem

Solve the problem using simplex-method.

$$F = -14X_1 + 10X_2 \rightarrow min$$
$$-21X_1 + 14X_2 \geq -800$$
$$-5.3X_1 + 12X_2 \leq 100$$
$$5.3X_1 + 9X_2 \leq 60$$
$$X_j \geq 0$$

Solution

Multiplying objective function and the first constraint by (-1) we have:

$$F = 14X_1 - 10X_2 \rightarrow max$$
$$21X_1 - 14X_2 \leq 800$$
$$-5.3X_1 + 12X_2 \leq 100$$
$$5.3X_1 + 9X_2 \leq 60$$
$$X_j \geq 0$$

Let's transform all this inequalities into equalities. Let's take new variables X_5, X_6, X_7. We have:

$$F = 14X_1 - 10X_2 \rightarrow max$$
$$21X_1 - 14X_2 + X_5 = 800$$
$$-5.3X_1 + 12X_2 + X_6 = 100$$
$$5.3X_1 + 9X_2 + X_7 = 60$$
$$X_j \geq 0$$

We got an valid initial solution, because all variables are positive.

Step 1

The biggest coefficient of objective function is 14 at X_1. Solving equation-constraint of X_1 we have:

$$X_1 = \frac{800}{21} = 38$$
$$X_1 = -\frac{100}{5.3} = -19 = \infty$$
$$X_1 = \frac{60}{5.3} = 11$$

Choosing the lowest value we get the new basis variable:

$$X_1 = \frac{60}{5.3} - \frac{9}{5.3}X_2 - \frac{X_7}{5.3}$$

Step 2

Next we substitute new basis variable into mathematical model.

$$F = 158.48 - 33.772X_2 - 2.632X_7 \rightarrow max$$

$$X_5 = 562.28 + 35.658X_2 + 3.948X_7$$

$$X_6 = 159.996 - 8.999X_2 - 0.996X_7$$

$$X_1 = 11.32 - 1.698X_2 - 0.188X_7$$

All objective function coefficients are negative, so we stop computation. The answer is the following:

$$F_{max} = 158.48$$

$$X_1 = 11.32$$

$$X_2 = 0$$