Problem 1

Payoff matrix for two players A and B:

Player B

$$\text{Player A} \quad \begin{pmatrix} 6 & 3 & 8 & 5 & 9 \\ 6 & 5 & 7 & 6 & 6 \\ 2 & 1 & 5 & 4 & 7 \\ 4 & 4 & 3 & 8 & 8 \end{pmatrix}$$

Determine game price and optimal strategies for both players.

Solution.

For player A the first strategy is dominating the third strategy, so we remove the third strategy.

For player B the first strategy is dominating the fifth strategy.

As result we have a matrix:

$$A = \begin{pmatrix} 6 & 3 & 8 & 5 \\ 6 & 5 & 7 & 6 \\ 4 & 4 & 3 & 8 \end{pmatrix}$$

Let's consider player A strategies:

$$\min_{i=1} a_{ij} = \min\{6;3;8;5\} = 3$$

$$\min_{i=2} a_{ij} = \min\{6;5;7;6\} = 5,$$

$$\min_{i=3} a_{ij} = \min\{4;4;3;8\} = 3,$$

$\alpha = \max_{j} \min_{i} a_{ij} = \max\{3;5;3\} = 5$ is a lower game price.

So, the lower game price is α=5, and the player A for maximizing the minimal gain should choose the second of three strategies.

For player B:

$$\max_{j=1} a_{ij} = \max\{6;6;4\} = 6,$$

$$\max_{j=2} a_{ij} = \max\{3;5;4\} = 5,$$

$$\max_{j=3} a_{ij} = \max\{8;7;3\} = 8,$$

$$\max_{j=4} a_{ij} = \max\{5;6;8\} = 8$$

$\beta = \min_i \max_j a_{ij} = \min\{6;5;8;8\} = 5$ is the upper game price.

Player B should also choose the second strategy. $\alpha=\beta$, so this game has a saddle point. Game price is equal to 5. For player A the optimal is second strategy of three, for player B the second of four strategies is optimal.

Problem 2

The firm has constructed 6 business-plans $(X_1, X_2, X_3, X_4, X_5, X_6)$. There are five cases depending exterior conditions $(Y_1, Y_2, Y_3, Y_4, Y_5)$. For every variant of business-plan (X_i $(i = \overline{1,6}$)and situation Y_j $(j = \overline{1,5})$ there is a computed profit, given in the table.

Business-plan	Exterior situation				
	Y_1	Y_2	Y_3	Y_4	Y_5
	Profit, millions $				
X_1	1.0	1.5	2.0	2.7	3.2
X_2	1.2	1.4	2.5	2.9	3.1
X_3	1.3	1.6	2.4	2.8	2.1
X_4	2.1	2.4	3.0	2.7	1.8
X_5	2.4	2.9	3.4	1.9	1.5
X_6	2.6	2.7	3.1	2.3	2.0

Choose the best business-plan or combination of developed plans.

Solution.

The payoff matrix of the game is given above. It's easy to see there are no dominating strategies in this game.

Next we compute:

α = *max* {*min*(l.0; 1.5; 2; 2.7; 3.2); *min*(l.2; 1.4; 2.5; 2.9; 3.l); *min*(l.3; 1.6; 2.4; 2.8; 2.l); *min* (2.1; 2.4; 3; 2.7; 1.8); *min*(2.4; 2.9; 3.4; 1.9; 1.5);*min*(2.6; 2.7; 3.1; 2.3; 2)} = *max*{l.0; 1.2; 1.3; 1.8; 1.5; 2} = 2,

also

β = *min* (*max*(1.0; 1.2; 1.3; 2.1; 2.4; 2.6); *max*(l.5; 1.4; 1.6; 2.4; 2.9; 2.7*);* *max*(2;2.5;2.4;3;3.4;3.l); *max*(2.7;2.9;2.8;2.7;l.9;2.3);*max*(3.2;3.l;2.l;l.8;l.5;2)}= *min*{2.6;2.9;3.4;2.9;3.2} = 2.6.

So, α≠β, that means there is no saddle point in this game, so we need to transform this problem to problem of linear optimization:

$$\min Z = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$$

with conditions:

$$\begin{cases} t_1 + 1.2t_2 + 1.3t_3 + 2.1t_4 + 2.4t_5 + 2.6t_6 \geq 1; \\ 1.5t_1 + 1.4t_2 + 1.6t_3 + 2.4t_4 + 2.9t_5 + 2.7t_6 \geq 1; \\ 2t_1 + 2.5t_2 + 2.4t_3 + 3t_4 + 3.4t_5 + 3.1t_6 \geq 1; \\ 2.7t_1 + 2.9t_2 + 2.8t_3 + 2.7t_4 + 1.9t_5 + 2.3t_6 \geq 1; \\ 3.2t_1 + 3.1t_2 + 2.1t_3 + 1.8t_4 + 1.5t_5 + 2t_6 \geq 1; \end{cases}$$

$$t \geq 0 \quad (i = \overline{1,6})$$

To solve this we use Excel Solver package. The optimal solution is t_2=0.11; t_6= 0.34. Optimal solution for initial problem is: $x_2^* = 0.24$; $x_6^* = 0.76$. The price of the game is υ= 2.264.