

Problem

For a given curve find the sphere that is tangent to the curve with the third order.

Solution

Let $r = r(s)$ is the parameterization of the curve. Sphere equation:

$$(r - a)^2 = R^2$$

where a is the center of the sphere and R is radius. Substituting $r = r(s)$ to this equation and differentiating this we got:

$$(r - a)\tau = 0$$

$$(r - a)k_1\nu + 1 = 0$$

$$(r - a)(k_1'\nu - k_1^2\tau - k_1k_2\beta) = 0$$

This implies

$$(r - a)k_1k_2\beta + \frac{k_1'}{k_1} = 0$$

So

$$(r - a)\tau$$

$$(r - a)\nu = -\frac{1}{k_1}$$

$$(r - a)\beta = -\frac{k_1'}{k_1^2k_2}$$

This implies

$$R = |r - a| = \sqrt{\left(\frac{1}{k_1}\right)^2 + \left(\frac{k_1'}{k_1^2k_2}\right)^2}$$

$$a = r + (a - r) = r + \frac{\nu}{k_1} + \frac{\beta k_1'}{k_1^2k_2}$$