Problem

Let $X = \{x_1; x_2; x_3\}$ is the universal set. Fuzzy set A is given by function given in the table below.

x	$\mu_A(x)$
x_1	0.4
x_2	0.6
x_3	1.0

Fuzzy map $\phi: X \to Y$ is given by table $\mu_\phi(x,y): X \times Y \to [0;1]$.

	y_1	y_2	y_3	y_4	y_5
x_1	0.7	1.0	0.1	0.3	0.6
x_2	0.7	0.4	0.7	0.5	0.6
x_3	0.3	0.4	0.1	0.2	1.0

(i)

Find the image of the fuzzy set A by $\mu_\phi(x,y)$.

(ii)

Find the preimage A^* of the fuzzy set B^* defined by the table

y	$\mu_{B^*}(y)$
y_1	0.6
y_2	0.4
y_3	0.6
y_4	0.5
y_5	1.0

Solution

(i)

Let $M(x,y) = \min\{\mu_A(x), \mu_\phi(x,y)\}$. Let's build the table

x	$M(x,y_1)$	$M(x,y_2)$	$M(x,y_3)$	$M(x,y_4)$	$M(x,y_5)$
x_1	0.4	0.4	0.1	0.3	0.4
x_2	0.6	0.4	0.6	0.5	0.6
x_3	0.3	0.4	0.1	0.2	1.0
$\mu_B(y)$	0.6	0.4	0.6	0.5	1.0

The last row of the table defines the set $B = \phi(A)$.

(ii)

Let's build

$$X_0 = \left\{x \in X \middle| \exists y \in Y, \mu_\phi(x,y) > \mu_{B^*}(y)\right\} = \{x_1, x_2\}$$

We see that $x_3 \notin X_0$.

So $\mu_{A^*}(x_3) = 1$. For x_1, x_2 let's build sets

$$N_{x_1} = \{y \in Y | \mu_\phi(x_1, y) > \mu_B(y)\} = \{y_1, y_2\}$$

$$N_{x_1} = \{y \in Y | \mu_\phi(x_2, y) > \mu_B(y)\} = \{y_1, y_3\}$$

Using the table

$$\mu_{A^*}(x_1) = \min\{0.3, 0.4\} = 0.3$$

$$\mu_{A^*}(x_2) = \min\{0.3, 0.7\} = 0.3$$

Thus the resulting set is:

x	$\mu_{A^*}(x)$
x_1	1
x_2	0.3
x_3	0.3