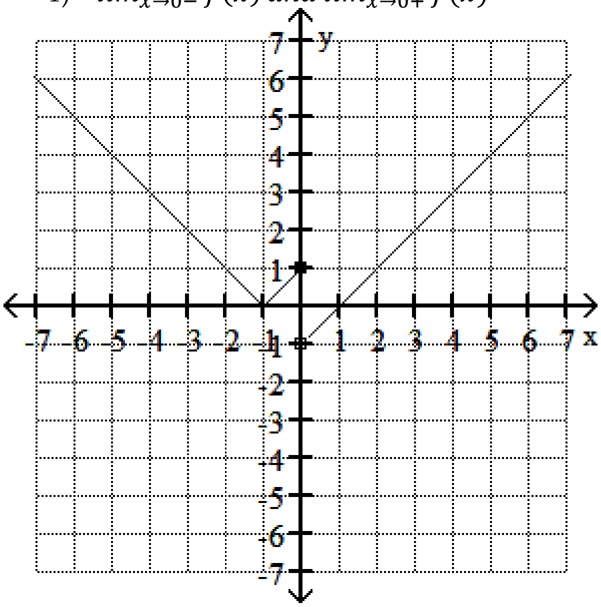
Decide whether the limit exists. If it exists, find its value.

1) $lim_{x \to 0-} f(x)$ *and* $lim_{x \to 0+} f(x)$



From picture we can see that function have one break in the point at $x = 0$ and equal to $f(0) = 1$, but function is continuous for any $\forall x, x \in (-\infty, 0]$. So

$$\lim_{x \to 0-} f(x) = 1$$

But $\forall \varepsilon, 0 < x < \varepsilon,\ f(x) = x - 1 < -1 + \varepsilon$, so

$$\lim_{x \to 0+} f(x) = -1$$

Find an equation of the tangent line at the indicated point on the graph of the function.

2) $s = h(t) = t^3 - 9t + 5, (t, s) = (3,5)$

The equation of tangent line is equal to next:

$s - s_0 = k(t - t_0)$, where (s_0, t_0) is point of tangency, and k – tangent, that is equal to derivative at this point.

$h'(t) = 2t^3 - 9, \rightarrow h'(3) = 45$

So, the equation of tangent line is $s = 45(t - 3) + 5$

Find the derivative.

3) $f(x) = \frac{4}{\sqrt{x}} - \frac{6}{x} + \frac{8}{x^4}$
Transform the function

$$f(x) = \frac{4}{\sqrt{x}} - \frac{6}{x} + \frac{8}{x^4} = 4x^{-\frac{1}{2}} - 6x^{-1} + 8x^{-4}$$

Now find the derivative, using the rule $(x^a)' = ax^{a-1}$

$$f'(x) = \left(4x^{-\frac{1}{2}} - 6x^{-1} + 8x^{-4}\right)' = 4 * \left(-\frac{1}{2}\right)x^{-\frac{3}{2}} + 6x^{-2} - 4 * 8 * x^{-5} = -\frac{2}{x^{\frac{3}{2}}} + \frac{6}{x^2} - \frac{32}{x^5}$$

4) If the price of a product is given by $P(x) = \frac{1024}{x} + 1200$, where x represents the demand for the product, find the rate of change of price when the demand is 8.

To find the rate of change, we should to derivative of $P(x)$ at $x = 8$.

$$P'(x) = -\frac{1024}{x} \to P'(8) = -128$$

So, rate of change price at demand 8 is p=-128

5) Find the first and second derivatives of the function $g(t) = (4t - 5)^{\frac{3}{2}}$

$$g'(t) = \left((4t-5)^{\frac{3}{2}}\right)' = \frac{3}{2}(4t-5)^{\frac{1}{2}} * (4t-5)' = 6(4t-5)^{\frac{1}{2}}$$

$$g''(t) = \left(6(4t-5)^{\frac{1}{2}}\right)' = 6 * \frac{1}{2}(4t-5)^{-\frac{1}{2}}(4t-5)' = 12(4t-5)^{-\frac{1}{2}}$$

6) Sketch the graph of the function, indicating all critical points and inflection points. Apply the second derivative test at each critical point. Show the correct concave structure.

$$f(x) = x^3 - 3x^2 - 9x$$

At first, find first derivative and all solution of equation $f'(x) = 0$ will be points, suspected to extreme (critical points).

$$f'(x) = (x^3 - 3x^2 - 9x)' = 3x^3 - 6x^2 - 9 = 3(x^2 - 2x - 3)$$

Roots of $f'(x) = 0 = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$ are $x = -1$ *and* $x = 3$

So, critical points are $x = -1, f(-1) = 5$ *and* $x = 3, f(3) = -27$

To check they , find second derivative, and check value. If $f''(x) \neq 0$, point is critical.

Checks they

$$f''(x) = 3(x^2 - 2x - 3)' = 6x - 6 = 0$$

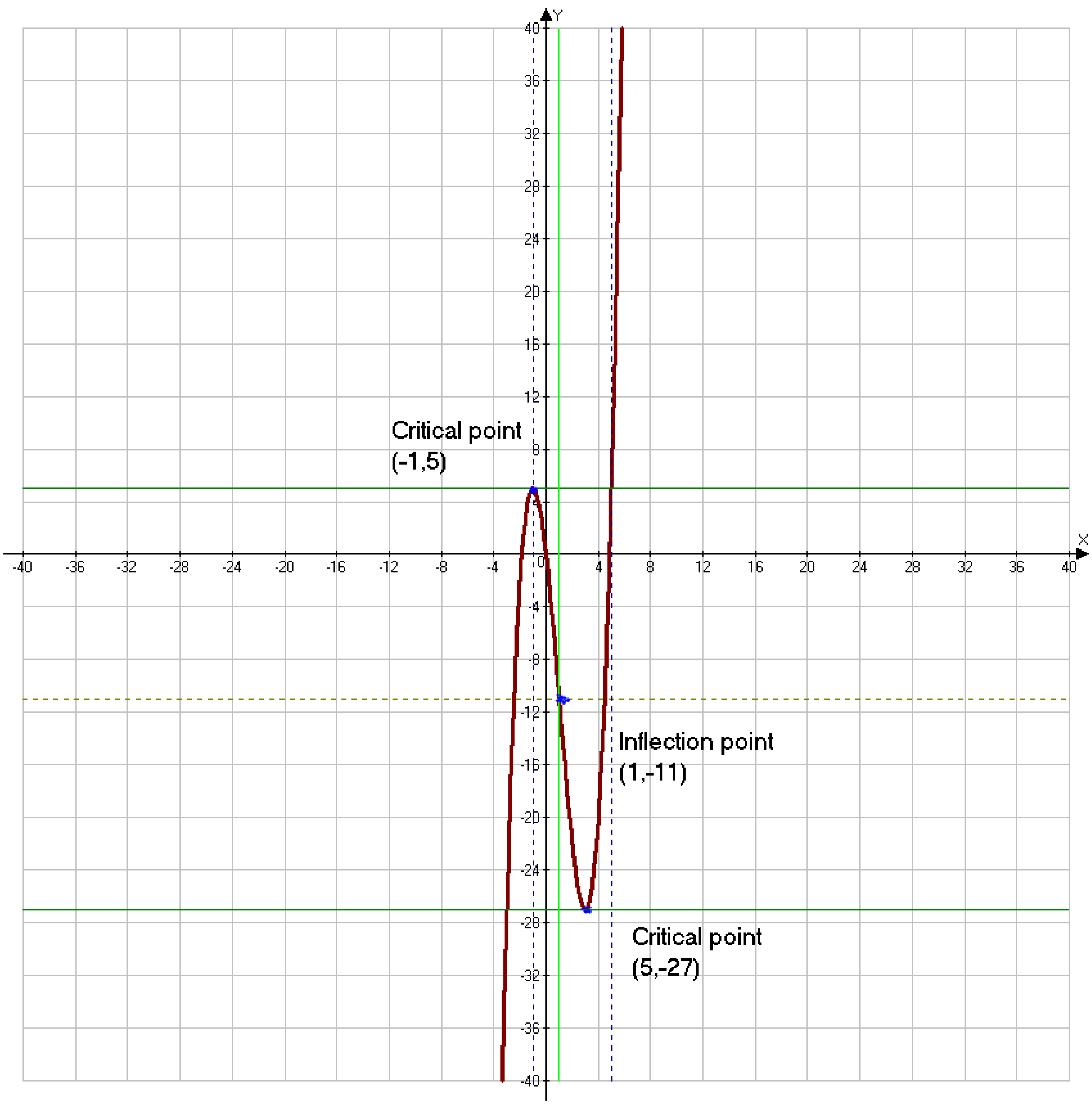
$f''(-1) = -12 \neq 0$

$f''(3) = 12 \neq 0$. So, both points are critical

Points, that can inflection points are roots of equation $f''(x) = 0$

$$f''(x) = 6x - 6 = 0 \Rightarrow x = 1 \; point\ of\ inflection.$$

Now sketch the graph



7) Sketch, by hand, the graph of f(x). Identify all extrema, inflection points, intercepts, and asymptotes. Show the concave structure clearly and note any discontinuities.

$$f(x) = \frac{x^2}{x-1}$$

To find derivation, use quotient rule $\left(\frac{u}{v}\right)' = \frac{u'v-v'u}{v^2}$

$$f'(x) = \left(\frac{x^2}{x-1}\right)' = \left(\frac{2x(x-1)-x^2}{(x-1)^2}\right) = \frac{(x-2)x}{(x-1)^2}$$

$$f''(x) = \left(\frac{(x-2)x}{(x-1)^2}\right)' = \frac{(2x-2)(x-1)^2-(x^2-2x)2(x-1)}{(x-1)^4} = \frac{2(x-1)^2-(x^2-2x)}{(x-1)^3} = \frac{2}{(x-1)^3}$$

So, we can see that root's of $f'(x) = 0$ are $x = 0$ *and* $x = 2$ and $f''(x) = 0$ have root's, so function have two critical points (0,0) and (2,4), and have not any inflection point.

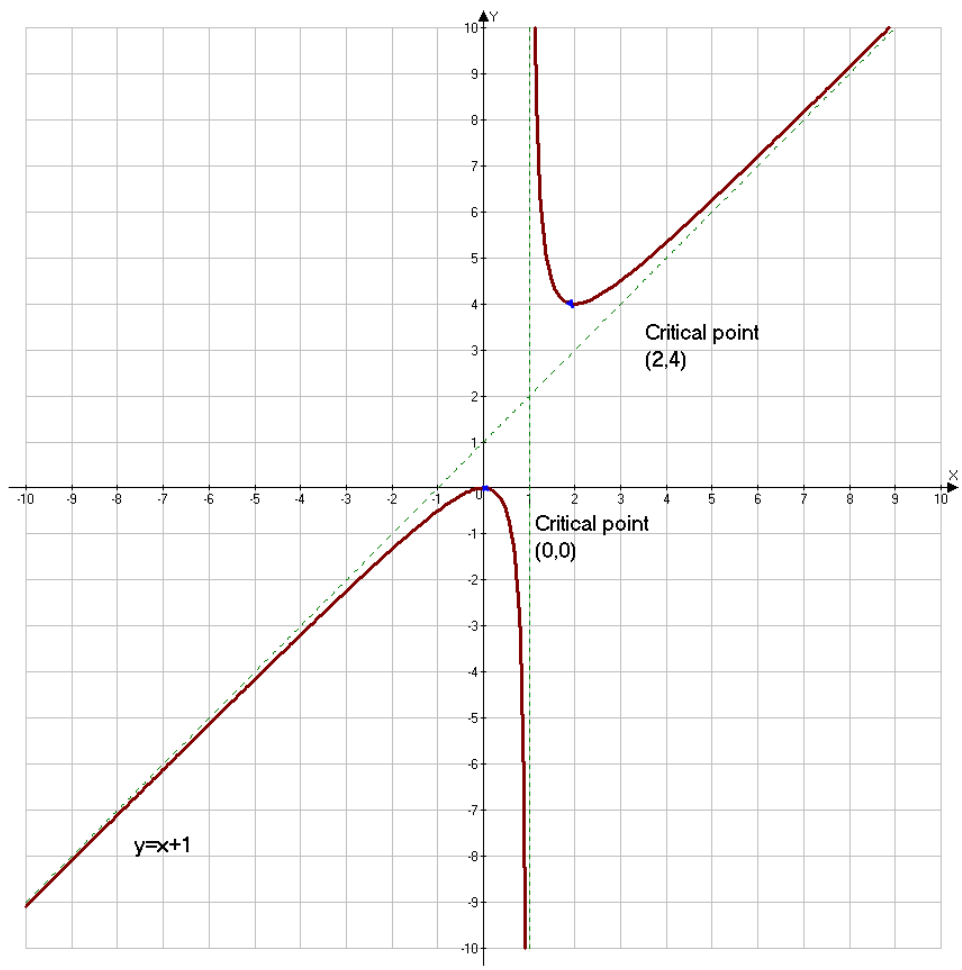
Also function have vertical asymptote $x = 1$, because denominator of the function is $x - 1 = 0$

Now find slope asymptote

$$f(x) = \frac{x^2}{x-1} = \frac{x(x-1)}{x-1} + \frac{x}{x-1} = x + \frac{x}{x-1}$$

We can see that when $x \to \infty, \frac{x}{x-1} \to 1, so\ f(x) \to x + 1$

So, horizontal asymptote will be equal to $y = x + 1$



8) The driver of a car traveling at 60 ft/sec suddenly applies the brakes. The position of the car is $s = 6ot - 3t^2$, t seconds after the driver applies the brakes. How many seconds after the driver applies the brakes does the car come to a stop?

$$s = 60t - 3t^2$$

Now find velocity

$$v(t) = 60 - 6t$$

When it comes to rest, then v=0, so $60 - 6t = 0 => t = 10$

Now distance traveled is

$$s = 60 * 10 - 3 * 10 * 10 = 300 \text{ ft}$$

Find the derivative

9) $y = \frac{e^{-x}+1}{e^x}$

At first, simplify the expression

$$y = \frac{e^{-x} + 1}{e^x} = e^{-2x} + e^{-x}$$

Now find derivative

$$y' = (e^{-2x} + e^{-x})' = -2e^{-2x} - e^{-x}$$

Find the derivative of the function.

10) $y = \ln(7 + x^2)$

$$y' = ((\ln p)' = \frac{1}{p}) = \frac{1}{x^2 + 7}(2x) = \frac{2x}{x^2 + 7}$$

11) Given the function $= \frac{(1+2x)^{\frac{3}{2}}}{(1+3x)^{\frac{4}{3}}}$, find $\frac{dy}{dx}$ by logarithmic differentiation.

For a function

$$y = f(x)$$

logarithmic differentiation typically begins by taking the natural logarithm, or the logarithm to the base e, on both sides, remembering to take absolute values

$$\ln|y| = \ln|f(x)|$$

After implicit differentiation

$$\frac{dy}{dx} = y \times \frac{f'(x)}{f(x)} = f'(x).$$

Multiplication by y is then done to eliminate 1/y and leave only dy/dx on the left:

$$\frac{dy}{dx} = y \times \frac{f'(x)}{f(x)} = f'(x).$$

The method is used because the properties of logarithms provide avenues to quickly simplify complicated functions to be differentiated. These properties can be manipulated after the taking of natural logarithms on both sides and before the preliminary differentiation. Used laws:

$$\log(ab) = \log(a)+\log(b), \qquad \log\left(\frac{a}{b}\right) = \log(a)-\log(b), \qquad \log(a^n) = n\log(a)$$

$$\ln y = \ln\left(\frac{(1 + 2x)^{\frac{3}{2}}}{(1 + 3x)^{\frac{4}{3}}}\right) = \frac{3}{2}\ln(1 + 2x) - \frac{4}{3}\ln((1 + 3x))$$

$$(\ln y)' = \frac{y'}{y} = \frac{3 * 2}{2(1 + 2x)} - \frac{4 * 3}{3(1 + 3x)}$$

$$y' = \frac{(1 + 2x)^{\frac{3}{2}}}{(1 + 3x)^{\frac{4}{3}}}\left(\frac{3}{1 + 2x} - \frac{4}{1 + 3x}\right)$$

12) A certain radioactive element has a half-life of 12 minutes. At what time is the substance decaying at a rate of 3.466 grams per minute if there are 120 grams present initially?

The law of natural dacay

$$\frac{dy}{dx} = ky$$

The solution of differential equation is

$y(x) = y(0)e^{kx}$ So solve it for k

Where x = 12, and y(x)=60, y(0)=120

So $60 = 120e^{k*12}$

$$12 * k = \ln\left(\frac{1}{2}\right)$$

$$k = -\frac{\ln 2}{12} \approx -0.05776$$

Going back to the law of natural decay

$\frac{dy}{dx} = ky$ =>$y = \frac{dy}{dx} * \frac{1}{k} = -3.466 * \frac{-12}{\ln 2} = 60min$