Problem 1

Find the ellipse equation if its eccentricity equals 0.6, focal length equals 3, focuses are located on the x-axis symmetrically of the origin.

Solution

Let's write statement of the problem in form of system of equations:

$$\begin{cases} c = \sqrt{a^2 - b^2} = 3 \\ e = \dfrac{c}{a} = 0.6 \end{cases}$$

Let's solve this system.

From the second equation:

$$a = \frac{c}{e} = \frac{3}{0.6} = 5$$

Substituting a into the first equation.

$$\sqrt{a^2 - b^2} = \sqrt{25 - b^2} = 3$$

$$b = 4$$

So ellipse equation is

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Problem 2

Write the equation and determine the curve type, if the curve consists of such points that distance to point $K(-2;0)$ is twice the distance to point $N(1;3)$.

Solution

Let M be a point on the curve. Then $MK = 2MN$. So

$$\sqrt{(x+2)^2 + y^2} = 2\sqrt{(x-1)^2 + (y-3)^2}$$

Transforming this equation:

$$x^2+4x+4+y^2=4x^2-8x+4+4y^2-24y+36$$

$$x^2-4x+y^2-8y+12=0$$

$$(x-2)^2+(y-4)^2=8$$

This is the equation of the circle with radius $R=2\sqrt{2}$ and center at point $(2;4)$.

Problem 3

Suppose the curve γ consists of points M, such that ratio of the distance from P to F(3;2) and distance from M to the line $y=1$ equals $\sqrt{3}$. Set up the equation, reduce it to canonical form and determine the type of the curve.

Solution

Let the point M on the curve has coordinates (x,y).

Denote by N projection of point M to the line $y=1$. Then

$$\frac{MF}{MN}=\sqrt{3}$$

$$MF=\sqrt{(x-3)^2+(y-2)^2}$$

$$MN=|y-1|$$

Equation of the curve:

$$\sqrt{(x-3)^2+(y-2)^2}=\sqrt{3}|y-1|$$

Squared equation:

$$(x-3)^2+y^2-4y+4=3y^2-6y+3$$

$$(x-3)^2-2\left(y-\frac{1}{2}\right)^2=-\frac{3}{2}$$

$$\frac{(x-3)^2}{\frac{3}{2}} - \frac{\left(y-\frac{1}{2}\right)^2}{\frac{3}{4}} = -1$$

$$\frac{\left(y-\frac{1}{2}\right)^2}{\frac{3}{4}} - \frac{(x-3)^2}{\frac{3}{2}} = 1$$

After changing variables

$$\begin{cases} x = Y \\ y = X \end{cases}$$

equation transforms to canonical form:

$$\frac{\left(X-\frac{1}{2}\right)^2}{\frac{3}{4}} - \frac{(Y-3)^2}{\frac{3}{2}} = 1$$

This is the equation of the hyperbola with center $\left(\frac{1}{2};3\right)$; real semiaxis $a = \frac{\sqrt{3}}{2}$; imaginary semiaxis $b = \sqrt{\frac{3}{2}}$ and focuses located on the line $X = \frac{1}{2}$.